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A PRELIMINARY EVALUATION OF THE ATCRBS SIGNAL FORMAT FOR THE BC--ETC(U)

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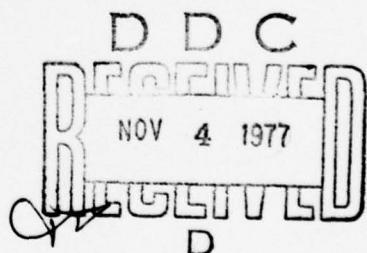
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A PRELIMINARY EVALUATION OF THE ATCRBS SIGNAL FORMAT FOR THE BCAS DATA LINK



AUGUST 1977

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15. Abstract The evaluation of the integrity of the ATCRBS signal format for the BCAS data link was based on measurements of the actual RF environment today, simulations of sophisticated signal processors, and basic calculations.			
16. The conclusions reached by the task force all relate to achieving a high integrity data link tailored to the BCAS application and were derived from tests run on the DABS ground-based reply processor-they are the following:			
<ul style="list-style-type: none"> (1) A data link with a high degree of error protection coding is essential. (2) Multiple transmissions - itself a form of coding - is essential. (3) A two-way data link is highly desirable from the point of view of the coordination logic. 			
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1. INTRODUCTION

The beacon-based collision avoidance system (BCAS) is an airborne equipment which senses proximity of other aircraft equipped with transponders and altitude encoders; if the other aircraft is deemed to be a threat, a collision avoidance maneuver is posted to the pilot. If the other aircraft is similarly equipped with BCAS, it too, will be calculating an escape maneuver. It becomes a key element of the system design to ensure that these maneuvers are complementary rather than conflicting (one climb, one dive is complementary; both climb is conflicting).

To ensure the proper coordination to give complementary maneuvers, some sort of air-to-air data-link communication is required. Since the coordination comes only when a threat has already been detected and a last-minute maneuver is about to commence, the integrity of the link is of vital importance. The most appropriate data link protocol for this application requires a complete round-trip communication to ensure that the other aircraft's intent is known at the time that a decision is required.

Over the past several years the signal format for the BCAS data link has received some attention. An early attempt to use the ATCRBS signal format was proposed by G. B. Litchford in a passive-mode application of several non-used bits in the ATCRBS reply (e.g.,

the X bit in the mode A and mode C replies). Subsequently others proposed combining a DABS and an ATCRBS mode in BCAS, and actively using the available DABS data link, known for its high link reliability. A recent proposal by G. B. Litchford offers several variations of active interrogations using the ATCRBS format instead of the DABS format. An evaluation of these latter ATCRBS techniques is the subject of this report.

To assess the performance of the ATCRBS signal format for use as the BCAS data link, a short study was undertaken by the FAA's Office of Systems Engineering Management. A task force was established with the following members participating:

Dr. E. J. Koenke	OSEM FAA, Chairman
Dr. P. M. Ebert	MITRE METREK
Mr. W. H. Harman	MIT Lincoln Laboratory
Mr. N. A. Spencer	MITRE METREK
Dr. A. Weinberg (part time)	MITRE METREK

The study was confined to a technical evaluation of the performance of the data link. Other non-technical factors--political, economic, and international--were not treated in this study.

The report will discuss the basis for the evaluation, establish a data base from which estimates will be made, show the results of

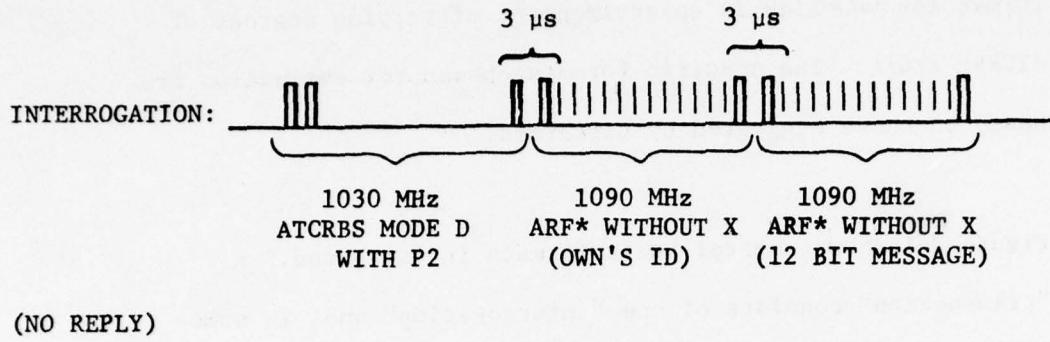
calculations for several ATCRBS formats as well, as for the DABS format, and, finally, summarize the conclusions.

2. METHOD OF APPROACH

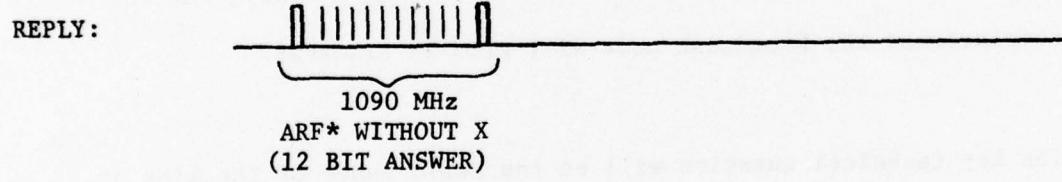
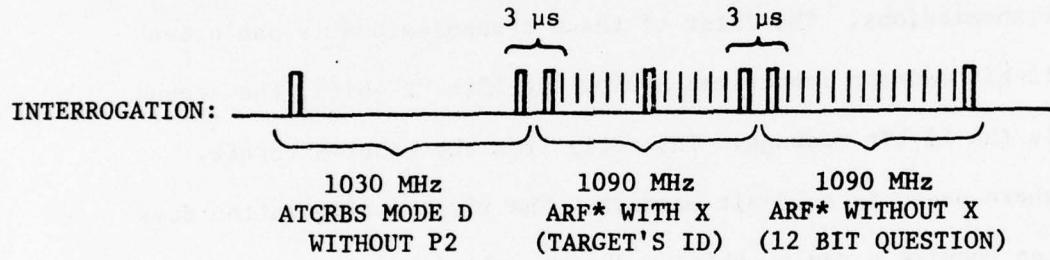
The point to be evaluated here is the performance of the ATCRBS format for data link in an environment of varying degrees of ATCRBS fruit. The specific formats chosen for evaluation are based on those suggested by Litchford.

Figure 2-1 shows several basic formats investigated. A "transaction" consists of one "interrogation" and, in some cases, one "reply". The interrogation consists of one 1030 MHz mode D transmission followed in about 3 μ s by two 1090 MHz transmissions. The first of these transmissions is one's own 12-bit address (sometimes adding the 13th "X" bit), the second is the 12-bit message. The reply from the other aircraft, when needed, is a 12-bit message. The first configuration does not require a reply--this is the so-called broadcast mode. As noted earlier, BCAS coordination should include a reply, but for completeness the broadcast mode will also be assessed.

The key technical question will be the performance of the link at 1090 MHz, where ATCRBS fruit could be a large and growing source of RF interference. The 1030 MHz link also is a factor in reliability, but its effect is considerably less than that of the 1090 MHz channel. This study considers only the latter.



(a) BROADCAST



(b) INTENT TRANSFER

* ARF DENOTES AN ATCRBS REPLY FORMAT.

NOTE: ERROR CONTROL CODING NOT USED.

FIGURE 2-1
POSSIBLE FORMATS FOR THE ATCRBS DATA LINK

Interference from fruit must be considered from two points of view, namely: the probability of getting the desired message to its destination; and the probability of getting a wrong message through. In a severe interference environment a single transaction has both a low probability of success and a relatively high probability of error (that is, of being interpreted erroneously). To overcome these consequences, the authors have computed the results when several strategies of multiple transmissions are used. One case would be to send the message 10 times, and to require the reception of identical messages for at least, say, 3 of these times. This will both increase the probability of successfully delivering the message, and decrease the probability of getting a false answer (e.g., receive a "climb" message when a "dive" message was sent).

The ability of the RF environment to introduce these problems depends not only on the fruit rate, but also on its power level relative to that of the aircraft with which we are trying to communicate. Thus, some assumptions must first be made about a typical target aircraft. This will be discussed in the next section.

3. BASIS OF ASSESSMENT

In the case considered, the air-to-air range is 5 nmi. This corresponds to the geometry at a time 30 seconds prior to collision in a head-on encounter of two 300 knot aircraft. Under nominal conditions, the received 1090 MHz power at this point is -61 dBm (see Table 3-1). Allowing 6 dB for power deviations from nominal, to account for deviations in transmitters, cabling, and aircraft antenna patterns, results in a worst-case power level of -67 dBm. According to the data in Reference 1, this power deviation allowance would be sufficient in almost all encounters involving two aircraft both equipped with antenna diversity. We therefore adopt the -67 dBm power level as a fixed point of reference for the data link evaluation.

TABLE 3-1

AIR-TO-AIR RF POWER BUDGET
 (1090 MHz, 5 nmi, Nominal Conditions)

1. Transmitter Power	dBm	57	Nominal
2. Transmitter Cabling Loss	dB	3	Nominal
3. Transmitter Antenna Gain	dB	0	Nominal
4. Free Space Path Loss (5 nmi)	dB	112	
5. Receiving Antenna Gain	dB	0	Nominal
6. Receiving Cabling Loss	dB	3	Nominal
7. Received Power	dBm	-61	Nominal
8. Power Deviation Allowance	dB	6	
9. Worst Case Received Power	dBm	-67	

4. MEASUREMENTS OF THE RF ENVIRONMENT

To support analysis of interference in the various BCAS modes a measurement program had been undertaken at Lincoln Laboratory to determine the present level of fruit in today's environment. For this, the Airborne Measurement Facility (AMF) was equipped with a 1090 MHz receiver, and recordings were made of fruit rate and power levels in several locations.

The plot in Figure 4-1 shows the relation between traffic density (determined from ARTS III data) and fruit rate. The receiver sensitivity is typical of what is required of BCAS and very similar to present military air-by-air IFF interrogator receivers. The "calculation" line has been used in earlier work, and is based on earlier measurements from the DABS program and before. A spread of measurements, typical of moment-to-moment variations, is shown for the Boston area as measured on the AMF in July of 1977. Each measurement data point plotted gives the average fruit rate over a 10 second period. These points are plotted at an average density of 0.01 ATCRBS transponders per nmi² which is an approximation of the density in the Boston area taken from Reference 1. Measurements in New York and Los Angeles will be conducted in the next several months.

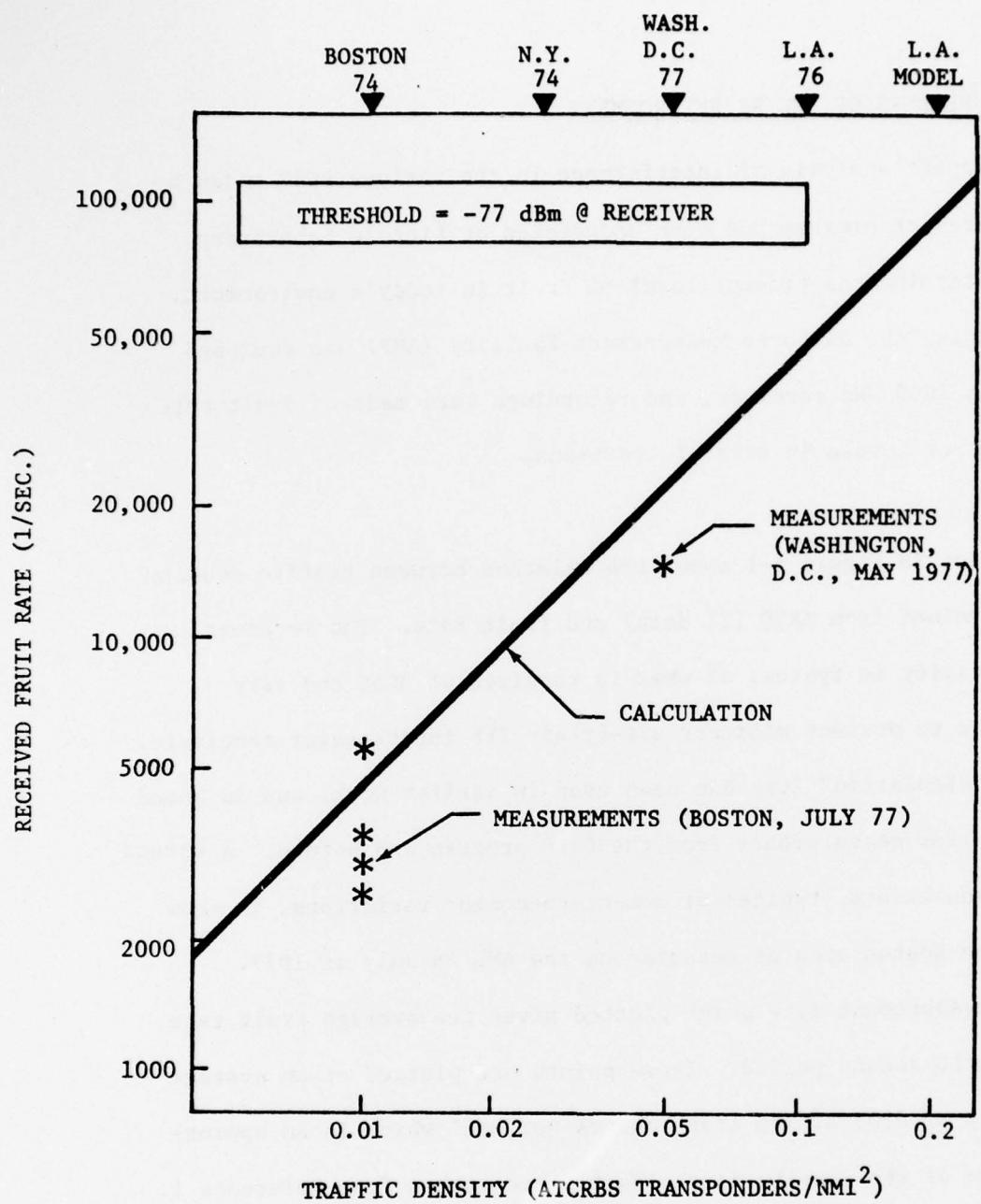


FIGURE 4-1
AIRBORNE FRUIT RATE

The point at 0.05 aircraft per nmi² is typical of aircraft replying Mode C at Washington, D.C. today, as measured with NAFEC's active-mode BCAS test bed. The measurements tend to confirm the calculation, and to lend credence to the model of interference.

5. REPLY PROCESSOR DATA

The ATCRBS reply processor considered for evaluation here is based on the sophisticated processor of a DABS ground station except as modified for BCAS by eliminating monopulse. The DABS reply processor is the present BCAS DABS-mode baseline design. To evaluate the basic capability of the reply processor to successfully detect and decode a single message in a fruit environment, use is made of an elaborate simulation at Lincoln Laboratory.

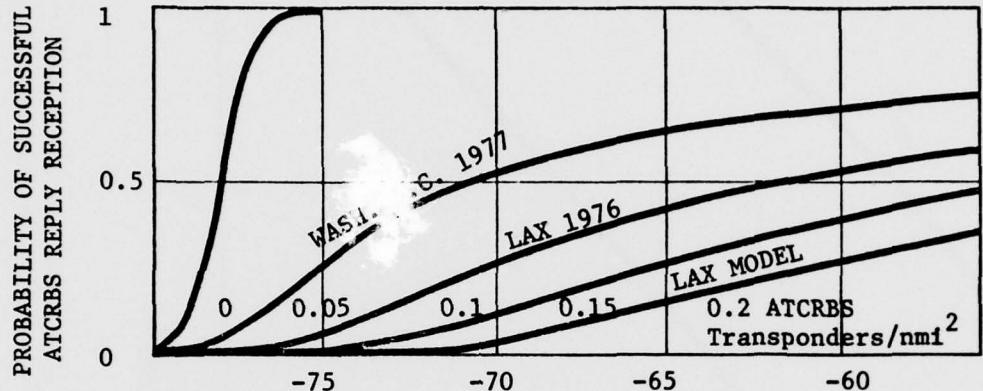
The probability of successful reception is computed by Monte Carlo method from a number of statistically independent trials. In each trial, the received signal is overlapped by a number of fruit replies, where the number is selected statistically and may be different from one trial to the next. The following are also selected statistically: the relative timing of each overlapping fruit reply, the received power level of each overlapping fruit reply, the pattern of 1's and 0's in each fruit reply, the RF phase of each pulse of each fruit reply, and the RF phase of each pulse in the signal reply. The combination of signal and interference is then added to a receiver noise background, bandpass filtered, envelope detected, and presented as the input to the ATCRBS or DABS reply processor. From this point on, the computer program carries out the reply processor functions exactly as specified (synchronous

sampling, leading edge detection, etc.), and thus is an algorithm for the hardware and software rather than a simulation.

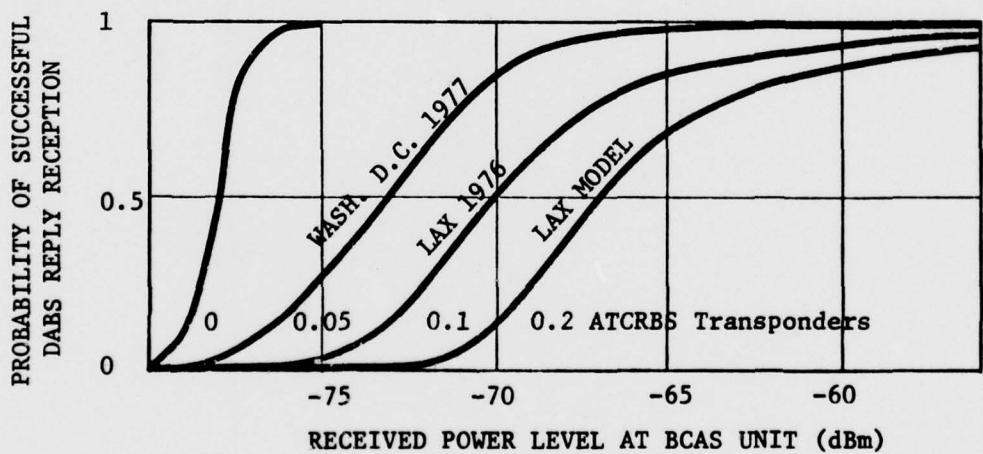
Figure 5-1 gives the ATCRBS reply processor performance as measured in the simulator. The results when using a DABS processor on a DABS signal format are also presented. In both cases, ATCRBS format and DABS format, ATCRBS fruit is viewed as the factor which ultimately limits air-to-air communications (Reference 1).

Figure 5-2 shows the probability of successfully receiving a single reply (12 bits) in the presence of ATCRBS fruit as generated at a typical high-density area by various densities of aircraft in that area. The plot of Figure 4-1 related density to fruit, as was discussed; the simulation measured probability of success as a function of fruit (Figure 5-1). The combined result is Figure 5-2.

During the course of this study it was realized that the ATCRBS performance could probably be improved by the addition of some form of dynamic desensitization to the reply processor. Whereas dynamic desensitization is generally regarded as being inappropriate in the ATCRBS reply processor of a DABS ground station, its use in BCAS may be warranted because of the severe fruit environment (which results from the omnidirectional receiving antenna). It was not possible within



(a) ATCRBS



(b) DABS

FIGURE 5-1

ATCRBS AND DABS REPLY PROCESSOR PERFORMANCE
IN A FRUIT ENVIRONMENT

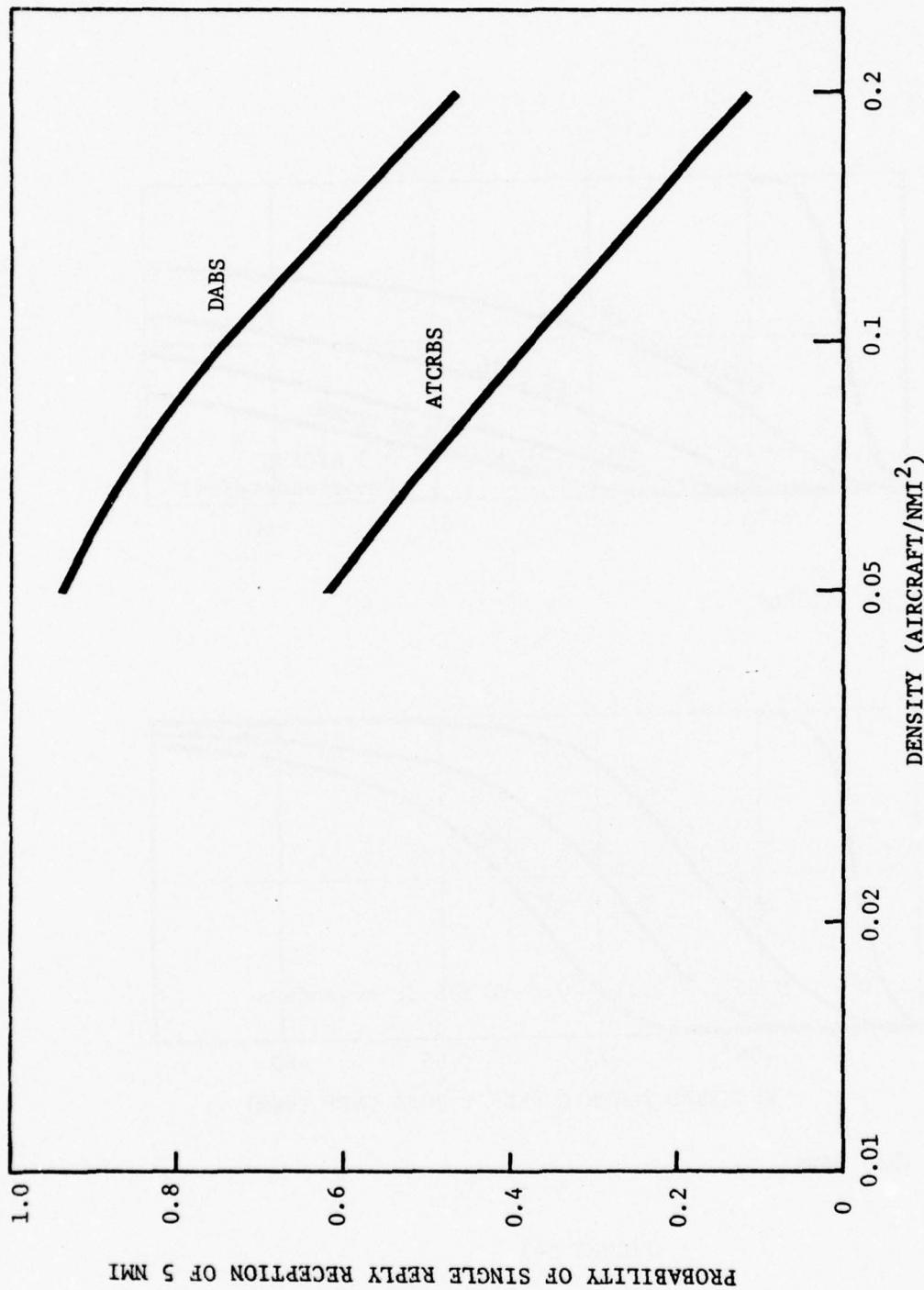


FIGURE 5-2
PROBABILITY OF RECEIVING A SINGLE REPLY VS. DENSITY

the short term of this study to assess such improvements in ATCRBS performance. Instead the ATCRBS evaluation is based on the existing data as given.

A further limitation in the available data as applied to this situation concerns synchronization. The data in Figure 5-1 applies to a single ATCRBS reply from which synchronization must be derived as a part of the reception process. In the ATCRBS data link, however, synchronization is already provided in the reception of the one or two ATCRBS reply formats contained within an "interrogation." Thus the ATCRBS performance might be slightly better than what is calculated, although this effect does not appear to be very significant.

6. DISCUSSION OF RESULTS

The theory and calculations of both the link reliability and the false message probability is provided in Appendix A. The curves resulting from that study are shown here as Figure 6-1 and 6-2. Each represents a different level of data-link capability.

The results for the technique which broadcasts intent and does not look for a reply are shown in Figure 6-1. The link reliability for the case labeled "1 out of 1" is approximately the square of the ATCRBS curve shown in Figure 5-2. That is, two messages must get through--the address and the intent. It is seen that for densities approaching today's busy airports the probability of success is fairly low; moreover, the probability of making an error by receiving the correct address and a false message is substantial (greater than 10 percent of the time, even for low density regions). The curves for 2, 3, and 4 out of 10 show the advantage in link reliability and correct-message probability gained by the burst transmission.* Also, the asterisks correspond to the link reliabilities below which the probability of receiving a wrong message is less than 10^{-4} (two, or more, out of 10, as the case may be). This factor is chosen as a baseline for undetected error, and is the measured value for the DABS undetected error at

* Two-out-of-ten means that out of ten transmissions the receiver would look for any two identical replies in the address and any two identical replies in the message. In Appendix A, this is called the "portion scheme".

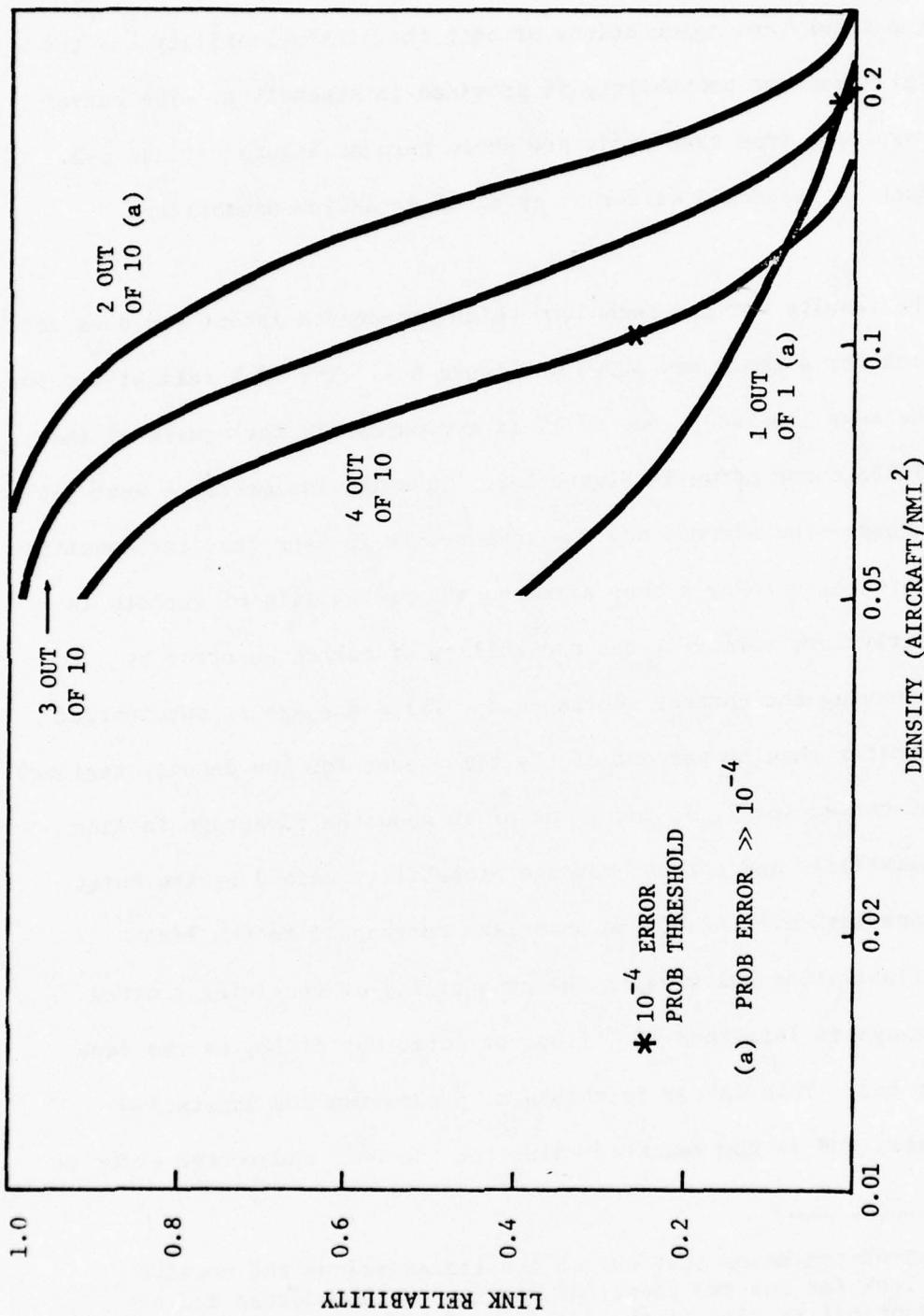


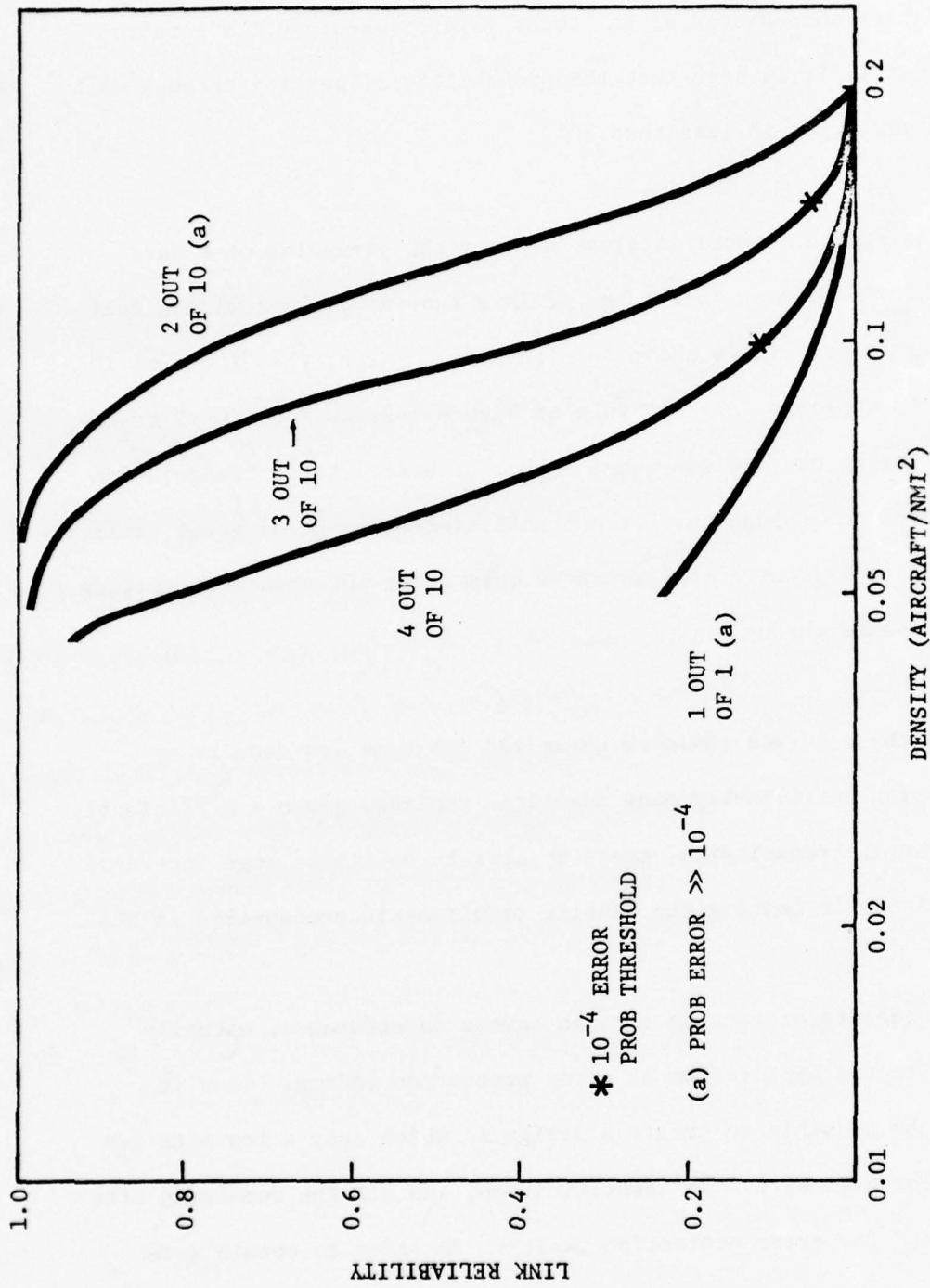
FIGURE 6-1
BCAS/ATCRBS DATA LINK PERFORMANCE - BROADCAST MODE

a density of 0.2 aircraft per nmi² (LAX model density). For densities encountered at LAX today (approximately 0.1 aircraft per nmi²), it is seen that the probability of getting through with this low error is less than 30%.

The next case is more interesting from the viewpoint of a data link for resolving intent, as it is a two-way link providing full coordination. It is shown in Figure 6-2. Here, the "1 out of 1" case is approximately the cube of Figure 5-2, as two ATCRBS transmissions go out and one comes back. As before burst transmission improves the situation, but not sufficiently to be of great utility--for LAX today there appears to be only about 10% chance of getting the message through.

While these curves could be optimized for some low density by requiring sufficiently many identical replies, given a sufficiently long burst transmission, there appears to be little hope for these techniques in meeting the density requirements encountered in BCAS.

The necessity of reading through severe interference, naturally leads to the application of error protection coding. Thus it might be possible to create a design in which only a few bits are used for data with mode identification, and all the remaining bits are used for error protection parity. In order to obtain some



appreciation of the value of this coding, the authors looked at the DABS signal format, which is an extreme, providing 32 information bits and 24 bits for parity.

Figure 6-3 shows the DABS capability. The curve labeled "1 out of 1" is simply a repeat of Figure 5-2. Since because of the coding a single DABS transaction has an undetected error probability of less than 10^{-4} for all densities under consideration, only a single receipt of the message is required. The effect of several transmissions is seen to be substantial, with "1 out of 10" providing essentially ideal performance. The DABS protocol, however, is simply to reinterrogate (within limits) until a reply is received, and then to stop. These curves show the value of reinterrogation and indicate that both multiple transmission and error-protection coding is necessary for the BCAS data link.

Using DABS results to evaluate the benefit of coding on the ATCRBS link is instructive, but the differences should also be recognized. In particular, the DABS modulation (PPM) is more easily distinguished from ATCRBS interference (PAM), and the DABS link uses the noiser 1090 MHz channel only one way, whereas the ATCRBS link uses it both ways. Nevertheless, it is reasoned that some form of multiple transmission and the use of additional bits for parity could improve the basic ATCRBS data link performance and conceivably overcome its limitations.

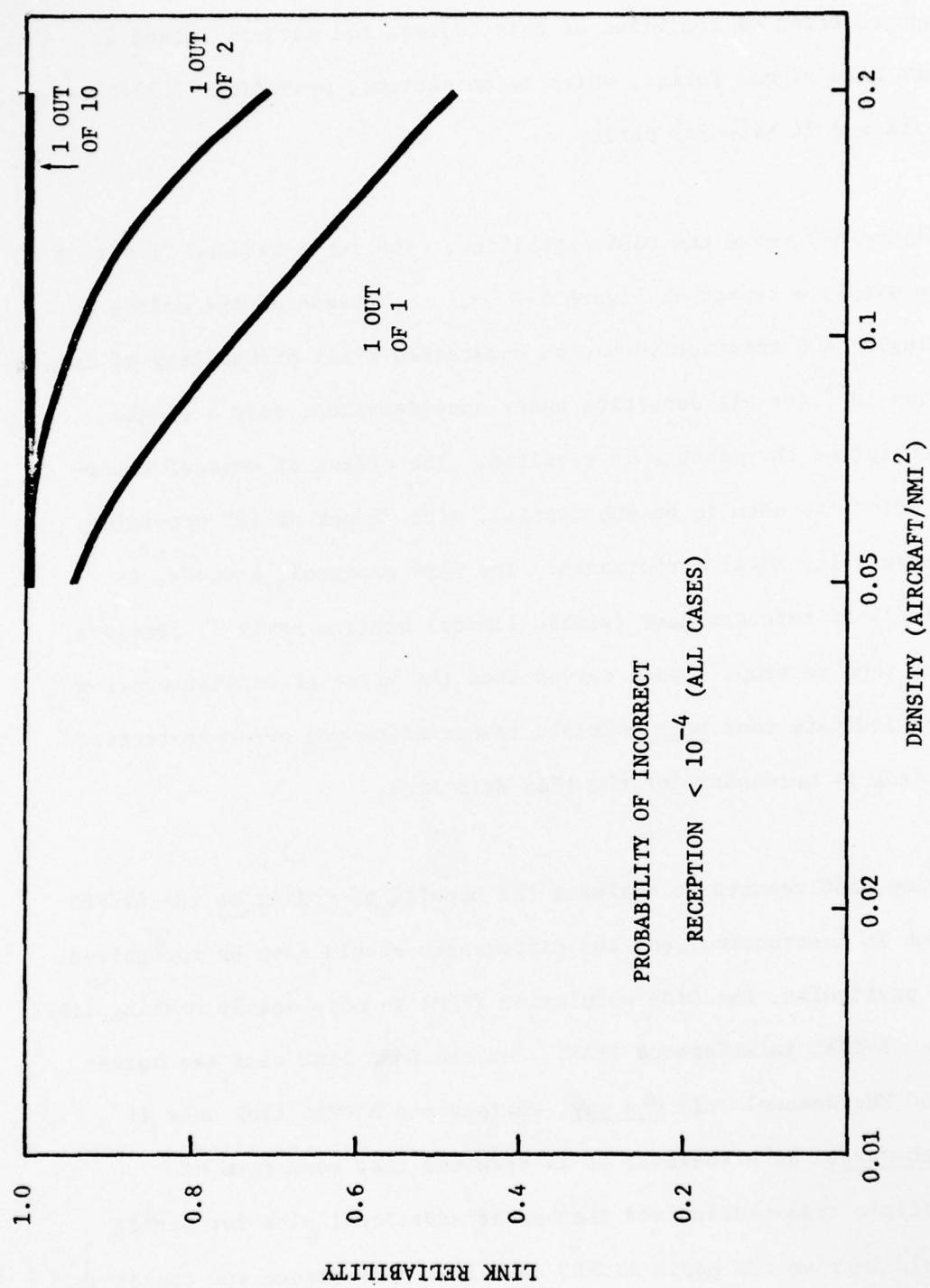


FIGURE 6-3

BCAS/DABS DATA LINK PERFORMANCE

7. CONCLUSIONS

The evaluation of the integrity of the ATCRBS signal format for the BCAS data link was based on measurements of the actual RF environment today, simulations of sophisticated signal processors, and basic calculations.

Several assumptions went into the assessment, the major ones being the following:

1. No dynamic desensitization (AGC) is applied to the receiver-processor. The processors used in ground systems do not employ this technique, nor are there any simulations presently available for evaluating it.
2. The simulation includes the need for obtaining synchronization from the ATCRBS signal, whereas synchronization comes from the Mode D transmission in the data-link application. Any differences are estimated to be small, at least so long as the link has reasonable reliability.
3. The probability of receiving the wrong message is a lower bound -- only single-bit errors were evaluated.

4. The probabilities of receiving any incorrect message were equally weighted; some incorrect messages are more important than others. This factor could be translated into changing the false message probability of 10^{-4} . This value was kept, however, because it is an established level of performance for DABS.

5. No consideration has been given here to the practical consequences of multipath and of the related factor of aircraft antenna shielding.

6. An ATCRBS error-protection-coded data link could be specified. The detailed design of such a link would take a significant amount of effort and was considered to be beyond the scope of this study.

The conclusions reached by the task force all relate to achieving a high integrity data link tailored to the BCAS application and were derived from tests run on the DABS ground-based reply processor--they are the following:

1. A data link with a high degree of error protection coding is essential.

2. Multiple transmissions--itself a form of coding--is essential.

3. A two-way data link is highly desirable from the point of view of the coordination logic.

APPENDIX A

LINK RELIABILITY AND FALSE MESSAGE PROBABILITY

The purpose of this appendix is to evaluate the probability of correctly and incorrectly transferring the intent of a BCAS aircraft, via each of several proposed schemes that are based on the ATCRBS Mode D data link.

A.1 Broadcast Technique

While this technique does not handle a full range of desired coordination, it is simple and will be evaluated as a prelude to the full round-trip technique.

The broadcast mode consists of two 1090 MHz blocks, the first with own ID and a zero in the X bit (i.e., 13-bits of information). The second block contains 12 bits of intent. To transfer the information correctly, both blocks must be correct. To increase the probability of success, especially in dense traffic, a burst of 10 transmissions is assumed, with the received data being accepted as valid if identical receptions occur at least K out of 10 times.

When multiple transmissions are available, there are two distinct ways to apply this "K out of 10" rule. In the first case, it would be required that the same sequence of 25 data bits are detected in each of at least K receptions; this will be termed the "whole" scheme. For the second possibility, the above restrictions are eased and it would only be necessary for the 13 ID bit, and 12 intent bit, sequences to be individually detected in at least K

receptions. In other words, a perfect match between K, 25 bit sequences, is not necessary in this procedure, which will be termed the "portion" scheme. The latter is desirable because it enhances link reliability, but, unfortunately, it also increases, the probability of an incorrect detection. These two schemes are now individually considered.

"Whole" Scheme

For the "whole" scheme, the probability of a successful single reception, P_o , is given by

$$P_o = P_{13} P_{12} \quad (A-1)^*$$

where:

$P_{13} = P_B (1-P_e)^{13}$, is the probability of detecting the 13 bit ID-and-X-bit correctly.

$P_{12} = P_B (1-P_e)^{12}$, is the probability of detecting the 12 bit intent message correctly.

P_B is the probability of a bracket decode on the message.

P_e is the bit error probability.

Thus, the formula for receiving K or more identical messages out of 10 identical messages is:

$$\begin{aligned} P_1 &= 1 - (1-P_o)^{10} && \text{for } K = 1 \\ P_2 &= P_1 - 10(1-P_o)^9 P_o && \text{for } K = 2 \\ P_3 &= P_2 - 45(1-P_o)^8 P_o && \text{for } K = 3 \\ P_4 &= P_3 - 120(1-P_o)^7 P_o && \text{for } K = 4 \end{aligned} \quad (A-2)$$

* P_o is the probability of success in a "1 out of 1" scheme.

On the other hand, to get the incorrect information (as opposed to not getting it), the ID data would have to be correct and the intent data incorrect in one or more positions.

The approach used here is to calculate the probability, on a single reception, of receiving a specific wrong message and then to calculate the probability of receiving that message at least K times out of 10 transmissions (assuming that the receiver knows the time interval in which the 10 transmissions will be made). Finally all possible erroneous messages are summed to yield the probability of incorrect reception. We have ignored the possibility that both the correct and an incorrect message will be decoded K or more times. An example calculation showed that the joint probability of these events is much smaller than the product of the marginal probabilities, and can safely be ignored.

Two other simplifications became apparent after several example calculations. The only incorrect messages that have any likelihood of achieving the K out of 10 threshold are those with only one bit wrong. Even though there are 5.5 times as many messages with two incorrect bits, they occur with such low probability that they do not contribute to the probability of error.

The last simplification is that completely random messages, such as fruit, do not produce an appreciable effect. They only increase the probability of receiving a given message by a very small amount, which is then even more suppressed by the K out of 10 rule.*

Using the above we write the approximate probability of correctly detecting an ID, followed by detecting a message with an error in the first bit, as,

$$P_F = P_{13} P_B (1-P_e)^{11} P_e \quad (A-3)$$

The probability of receiving this error pattern K or more times out of 10 tries is then,

$$\begin{aligned} P_{I1} &= 1 - (1-P_F)^{10} & K = 1 \\ P_{I2} &= P_{I1} - 10(1-P_F)^9 P_1 & K = 2 \\ P_{I3} &= P_{I2} - 45 (1-P_F)^8 P_1^2 & K = 3 \\ P_{I4} &= P_{I3} - 120 (1-P_F)^7 P_1^3 & K = 4 \end{aligned} \quad (A-4)$$

Identical probabilities are obtained if the second intent bit is in error, or the third, etc. Thus, the overall probability of getting a single bit in error is simply $12 P_{IK}$.**

* Based on this observation and the ones above, we may conclude that the probability of incorrect detection, to be calculated, is actually a lower bound.

** $12 P_F$ is the probability of an incorrect message reception in a "1 out of 1" scheme.

"Portion" Scheme

In this case we consider the probabilities of individually detecting the ID and intent bits. Using the above parameters, we obtain the following probabilities, which corresponds to those of (6-2):

$$\begin{aligned} P_K &= P_{AK} P_{BK} \\ P_{A2} &= 1 - (1-P_{13})^{10} - 10 (1-P_{13})^9 P_{13} \\ P_{A3} &= P_{A2} - 45(1-P_{13})^8 P_{13} \\ P_{A4} &= P_{A3} - 120(1-P_{13})^7 P_{13} \end{aligned} \quad (A-5)$$

The P_{BK} values are obtained by replacing P_{13} by P_{12} .

For an incorrect detection to occur the ID must still be detected correctly. If we thus define,

$$P_F' = P_B (1-P_e)^{11} P_e \quad (A-6)$$

which is the probability of detecting an intent message with a single error in a specific location, then the probabilities corresponding to (A-4) are here given by,

$$P_{IK}' = P_{AK} P_{IK} \quad (A-7)$$

where P_{IK} is given in (A-4), but with P_F replaced by P_F' . The overall probability of an incorrection detection is then $12 P_{IK}'$.

A.2 Intent Transfer

For this situation, one aircraft requests information from another that may be a potential threat. This is accomplished by having the former transmit as in the broadcast mode. Here, however, the 13 ID bits identify the aircraft interrogated,* with the following 12 bits specifying a question to be answered. The receiving aircraft would then reply, again over 1090 MHz, with a 12 bit message. As above "whole" and "portion" schemes may be considered.

"Whole" Scheme

Here, each time the receiving aircraft detects its identity, it responds in accordance with the detected question. Via the K out of 10 rule, then a successful transfer is accomplished if at least K out of 10 such round trip transmissions yield correctly detected forward and return data. The probability of success, P_K , is thus once again given by (A-2), but with P_o replaced by,

$$P_o = P_{13} P_{12}^2 \quad (A-8)$$

For an incorrect transfer, the forward ID must be detected correctly with either the question or reply detected incorrectly. It is assumed here that when an error occurs it does so in one of the two 12 bit messages, but not both.** Furthermore, only a single

* The 13th bit, the X bit, would here equal 1 to indicate that a reply is requested.

** The probabilities of multiple errors are neglected here.

bit error is assumed, as was the case for the broadcast mode.

Based on this description, then, the probability of incorrectly detecting a single, specific bit is given by,

$$P_F = P_{13} P_{12} P_B (1-P_e)^{11} P_e \quad (A-9)^*$$

The corresponding error probabilities, P_{IK} , are then given by (A-4). Finally, to obtain the overall error probabilities, the P_{IK} are multiplied by 24--as opposed to 12 for the broadcast mode--since the bit error can occur in any of the 12 forward or 12 return bits.

"Portion" Scheme

To implement the "portion" scheme, the receiving aircraft would first examine all 10 receptions and then apply the appropriate K out of 10 rule. If the detection rule is satisfied, the aircraft responds 10 times with the same 12 bit reply message. The probability of successful transfer is then given by

$$P_K = P_{AK} P_{BK}^2 \quad (A-10)$$

where these parameters are defined in (6-5). Also, the probability of an incorrect detection P_{IK}' , for a single, specific, bit error, is

$$P_{IK}' = P_{AK} P_{BK} P_{IK} \quad (A-11)$$

where P_{IK} is defined by (A-4), with P_F replaced by P_F' of (A-6).

The overall probability of incorrect detection is then 24 P_{IK}' .

* 24 P_F is the probability of an incorrect message reception in a "1 out of 1" scheme.

A.3 Performance Results

Performance results pertaining to the above schemes were computed as a function of aircraft density, ρ (aircraft/square mile). The bracket decode and bit error probabilities, P_B and P_e , respectively, were the underlying probabilities defined, and they were determined from Figure 5-2, together with other empirical data provided by Lincoln Laboratory (Reference 3). Table A-1 summarizes these empirical quantities and Table A-2 contains computed results, based on the analysis of the previous subsections. In Table A-2 P_c and P_I are the probabilities of correct and incorrect detections, respectively.

An examination of Table A-2 indicates that for a $P_I \approx 10^{-4}$ --which is the DABS performance level--the "portion" scheme generally outperforms its "whole" scheme counterpart. For this reason, only results pertaining to the former are plotted in Figures 6-1 and 6-2.

TABLE A-1
EMPIRICAL RESULTS

ρ	P_B	P_e
0.05	0.943	0.0336
0.10	0.739	0.056
0.15	0.659	0.084
0.20	0.456	0.112

TABLE A-2
COMPUTED PERFORMANCE RESULTS

ρ	$S_{C_H E_M E}$	BROADCAST						INTENT TRANSFER						INTENT TRANSFER						INTENT TRANSFER					
		1 OUT OF 1			2 OUT OF 10			3 OUT OF 10			4 OUT OF 10			1 OUT OF 1			2 OUT OF 10			3 OUT OF 10			4 OUT OF 10		
		P_c	P_I	P_c	P_I	P_c	P_I	P_c	P_I	P_c	P_I	P_c	P_I	P_c	P_I	P_c	P_I	P_c	P_I	P_c	P_I				
0.05	WHOLE	.378	.158	.939	.087	.795	.003	.562	7.1×10^{-5}	.237	.198	.725	.035	.434	7.7×10^{-4}	.194	1.1×10^{-5}								
	PORTION	.378	.158	.998	.227	.981	.013	.912	4.8×10^{-4}	.237	.198	.997	.454	.973	.026	.878	9.3×10^{-4}								
0.1	WHOLE	.129	.092	.378	.030	.129	6.2×10^{-4}	.031	8.0×10^{-6}	.048	.068	.080	.004	.010	3.2×10^{-5}	8.7×10^{-4}	$< 10^{-6}$								
	PORTION	.129	.092	.852	.211	.575	.010	.262	2.6×10^{-4}	.048	.068	.794	.394	.448	.016	.141	2.8×10^{-4}								
0.15	WHOLE	.046	.053	.082	.010	.011	1.2×10^{-4}	9.0×10^{-4}	10^{-6}	.011	.025	.005	5.6×10^{-4}	1.6×10^{-4}	2.0×10^{-6}	3.0×10^{-6}	$< 10^{-6}$								
	PORTION	.048	.053	.464	.141	.147	.004	.025	6.3×10^{-5}	.011	.025	.328	.199	.061	.004	.005	2.3×10^{-5}								
0.2	WHOLE	.010	.016	4.8×10^{-3}	9.7×10^{-4}	1.4×10^{-4}	3.0×10^{-6}	3.0×10^{-6}	$< 10^{-6}$.001	.004	6.1×10^{-5}	1.2×10^{-5}	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$	$< 10^{-6}$								
	PORTION	.010	.016	.076	.024	.006	2.3×10^{-4}	2.1×10^{-4}	10^{-6}	.001	.004	.015	5.1×10^{-4}	4.1×10^{-5}	4.0×10^{-6}	$< 10^{-6}$									

APPENDIX B

REFERENCES

1. "BCAS Quarterly Technical Letter," QTL-4-3, MIT Lincoln Laboratory, 20 July 1977.
2. "Design Concept for a Beacon-Based Airborne Collision Avoidance System." MIT Lincoln Laboratory, Working Paper, 42WP-5040, 3 February 1975.
3. "The DABS Mode of BCAS," briefing by MIT Lincoln Laboratory to Air Transport Association, 18 January 1977.